Part D	Problems 21-28 which require complete solutions.
Test time	120 minutes.
Resources	Digital resources, formula sheet and ruler.

Level requirements

The test consists of three written parts (Part B, Part C and Part D).
Together they give a total of 61 points consisting of
22 E-, 22 C- and 17 A-points.

Level requirements for test grades
E: 15 points
D: 24 points of which 7 points on at least C-level
C: 31 points of which 12 points on at least C-level
B: 41 points of which 5 points on A-level
A: 49 points of which 9 points on A-level

The number of points you can get for a complete solution is stated after each problem. You can also see what knowledge levels (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A- point.

For problems labelled *"Only answer is required"* you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures and show how you use your digital resources.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____

Date of birth: _____

Educational programme: _____

Part D: Digital resources are allowed. Write your solutions on separate sheets of paper.

21. The figure shows a complex plane where the number z has been marked.
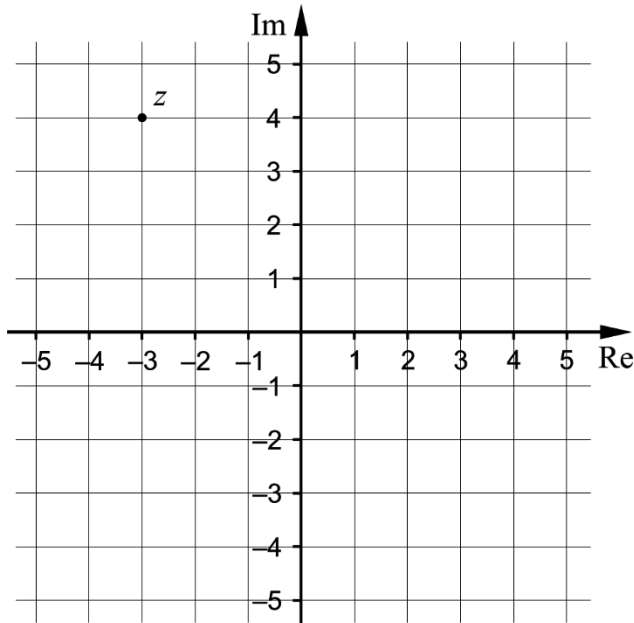

Determine the number z in polar form. (2/0/0)

22. One summer day in Pajala it was raining between 9:00 a.m. and 7:00 p.m. During these 10 hours the intensity of the rain was measured.

According to a simplified model the intensity of the rain is given by

$$y = x \cdot \sin \frac{\pi \cdot x}{10}$$

where y is the intensity of the rain in mm/h and x is the time in hours from 9:00 a.m. The model is assumed to be valid between 9:00 a.m. and 7:00 p.m.
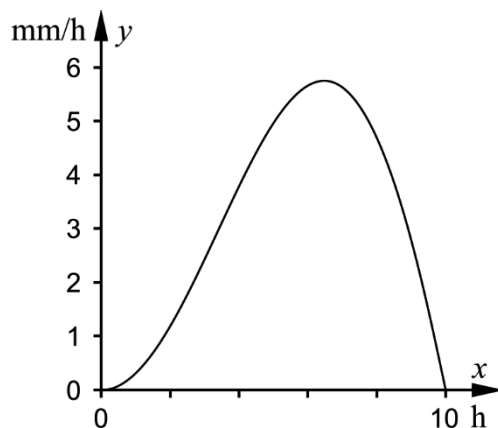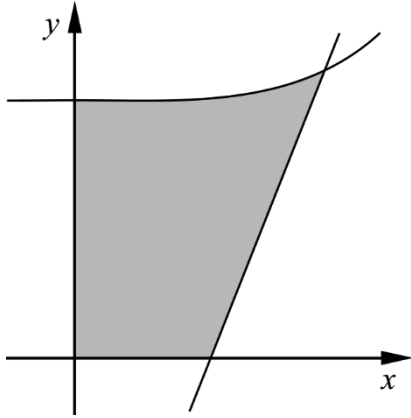

Calculate how many mm of rain in total fell during these 10 hours. (2/0/0)

23. The figure shows the graphs of the functions $f(x) = \dfrac{x^4}{4} + 1.5$ and

$g(x) = 4x - 2$

The shaded region in the figure is bounded by the two graphs of the functions and the positive coordinate axes.

Determine the area of the shaded region.
Give your answer to at least three significant figures. (2/1/0)

24. During a cloudless day with 12 hours of sunlight, the intensity I of the

sunlight can be approximated by $I = I_0 \sin^3\left(\dfrac{\pi x}{12}\right)$

where I_0 is the maximum intensity and x is the time in hours after sunrise.

a) Determine what percentage of its maximum intensity the sunlight has 3 hours after sunrise. (1/0/0)

A dermatologist recommends that sun protection is used if the intensity exceeds 50% of the maximum intensity.

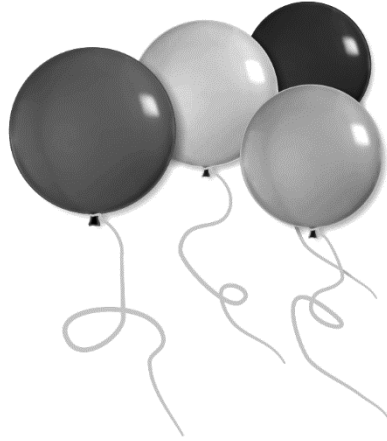b) Determine for how many hours sun protection should be used on this day according to the recommendation. (0/2/0)

3

25. It holds for the function f that $f''(x) = \cos x - \sin 2x$

At the point $(0,\ 1)$, the graph of the function f has the tangent $y = 2x + 1$

Find $f'(x)$. (0/3/0)

**26.** The party organisers Skoj & Ploj fill balloons with an air compressor.

The balloons can be considered to be spherical and each balloon should be filled to the volume 5.5 litres. The radius of the balloon increases at 3.5 cm/s at the time when the radius is 6.0 cm.

The air compressor fills the balloon with air smoothly so that the volume increases at a constant speed.

Determine how long it takes to fill a balloon which is empty at the beginning. (0/2/2)

27. Simone is a glass designer and has designed a vase that is 3 dm high. The shape of the vase can be described by the solid of revolution which appears when the shaded region in the figure is rotated around the x-axis. The shaded region is bounded by the curves $y_1 = \dfrac{\sqrt{2x+1}}{2}$, $y_2 = \dfrac{\sqrt{2x-0.5}}{2}$, the line $x = 3$ and the positive coordinate axes.
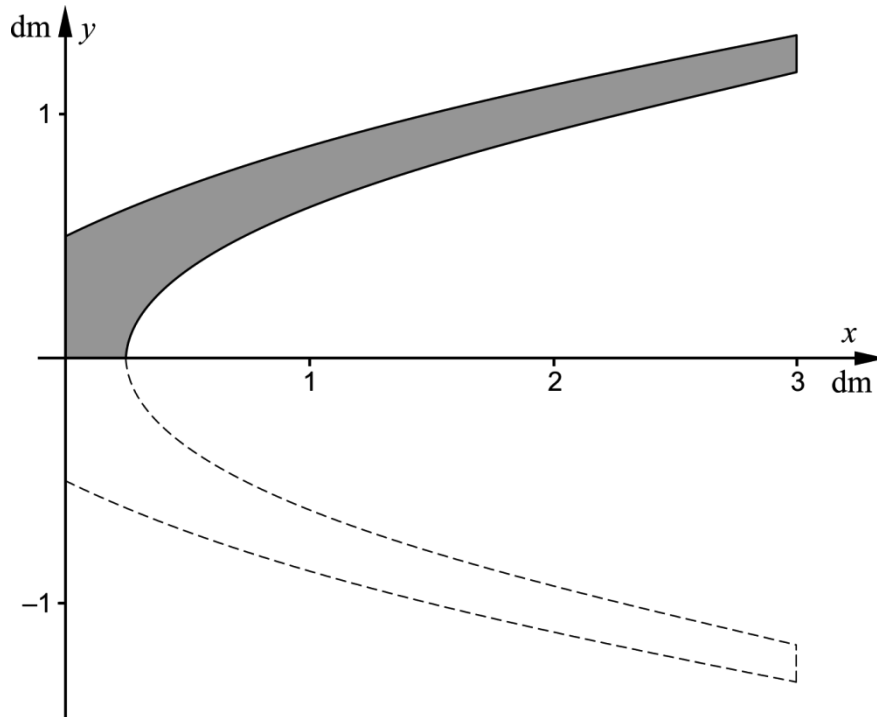
Simone wants to know how much frit she needs to produce the vase.

Calculate the volume of frit she needs. (0/1/2)

28. The function h is defined by $h(x) = (f(x))^2$.

Find $h''(0)$ for all functions f with the following properties:

- $f(0) = -1$
- $f'(0) = 3$
- $f''(0) = 2$ (0/1/3)