Part B	Problems 1-10 which only require answers.
Part C	Problems 11-16 which require complete solutions.
Test time	120 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 65 points consisting of 23 E-, 23 C- and 19 A-points.

Level requirements for test grades
E: 17 points
D: 26 points of which 8 points on at least C-level
C: 34 points of which 14 points on at least C-level
B: 44 points of which 6 points on A-level
A: 53 points of which 11 points on A-level

The number of points you can get for a complete solution is stated after each problem.
You can also see what knowledge levels (E, C and A) you can show in each problem.
For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled "*Only answer is required* " you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: ____

Date of birth: ____

Educational programme: ________________________________

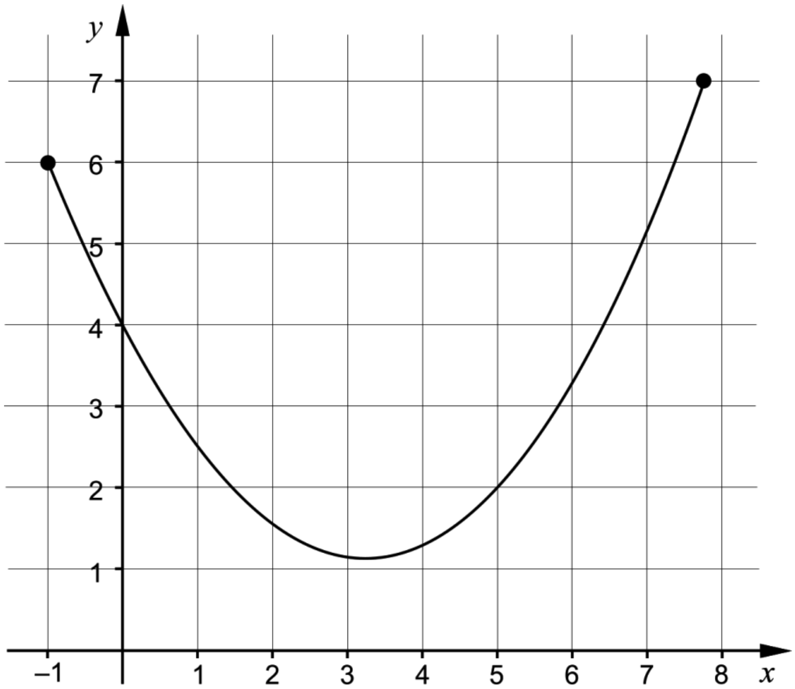
Part B: Digital resources are not allowed. *Only answer is required*. Write your answers in the test booklet.

1. For what value of x is the expression $\frac{2x}{x+4}$ not defined?

_______________________ (1/0/0)

2. Calculate the exact value of $\int_0^2 x^2 \mathrm{d}x$ _______________________ (1/0/0)

3. The figure shows the graph of a function that is defined on a closed interval.



In the figure, draw

a) a tangent with a gradient of 1. Label the tangent with the letter T. (1/0/0)

b) a secant line with a gradient of 1. Label the secant line with the letter S. (1/0/0)

4. Find $f'(x)$ if

a) $f(x) = 5x^3 - 8x^2 + 10$ $\qquad f'(x) =$ ____________________ (1/0/0)

b) $f(x) = \dfrac{3x + e^{-x}}{2}$ $\qquad f'(x) =$ ____________________ (0/1/0)

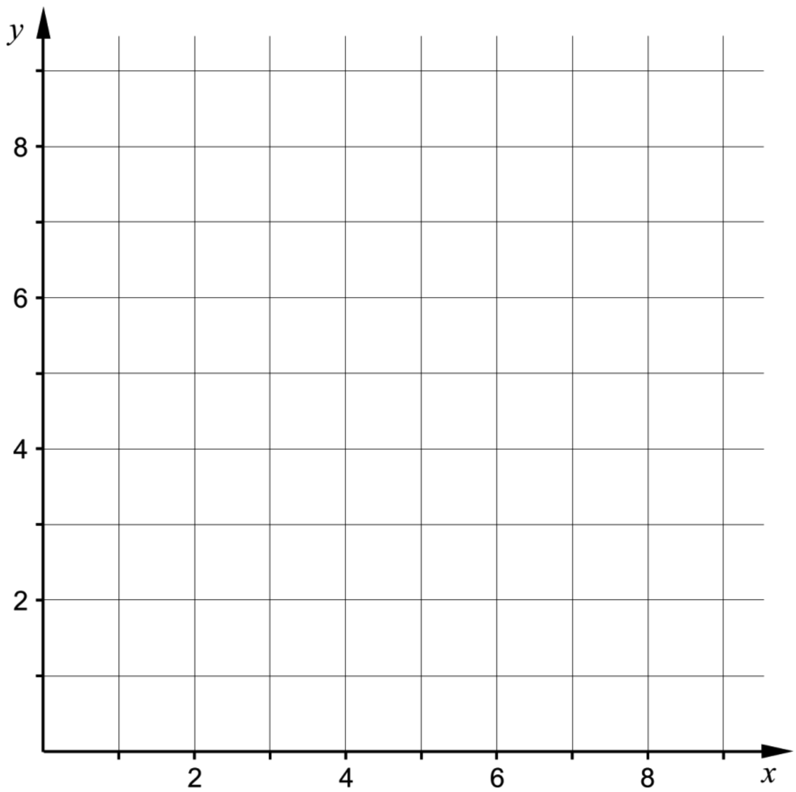
c) $f(x) = -\dfrac{2}{\sqrt{x}}$ $\qquad f'(x) =$ ____________________ (0/1/0)

5. Find the exact value of

a) $\sin 90° + \sin 150°$ ____________________ (1/0/0)

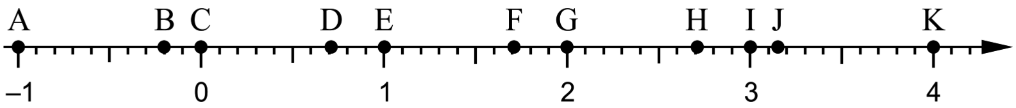
b) $\cos 240°$ ____________________ (0/1/0)

6. The function f is a *discrete* function.
It holds that $f(x) = x^2$ for $x = 1,\ 2$ och 3
Draw the graph of the function f in the coordinate system.



(1/0/0)

7. The points A – K are marked on the number line.

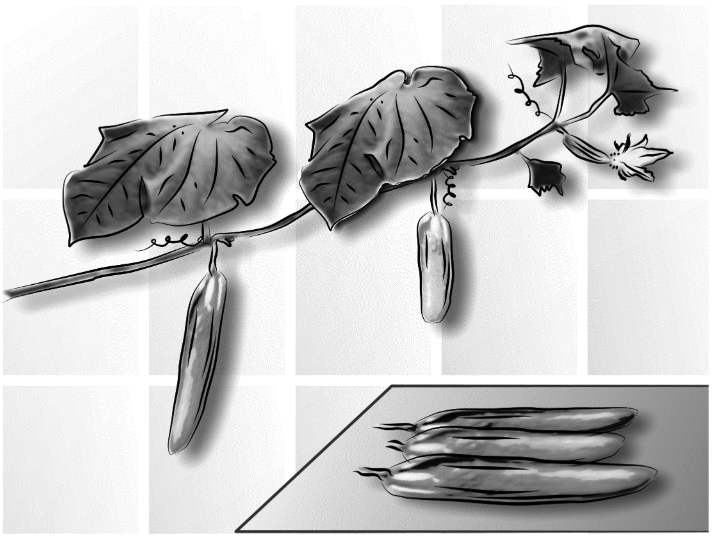


Determine which of the points A – K corresponds to the value of

a) $\ln e^2$ ______________________ (1/0/0)

b) $e - \ln 1$ ______________________ (0/1/0)

8. A cucumber farmer has investigated how the weight of a growing cucumber increases over time. She presents the result as a function $y = V(t)$, where $V(t)$ is the weight of the cucumber in hg and t is the time in weeks after the measuring was started.



What will she find out by calculating $V'(3)$?
Choose one of the alternatives A – E.

A. The weight of the cucumber in hg at the time 3 weeks.

B. The increase in weight of the cucumber in hg over 3 weeks.

C. The average increase in weight of the cucumber in hg/week over 3 weeks.

D. The time it takes for the weight of the cucumber to increase to 3 hg.

E. The increase in weight of the cucumber in hg/week at the time 3 weeks.

______________________ (0/1/0)

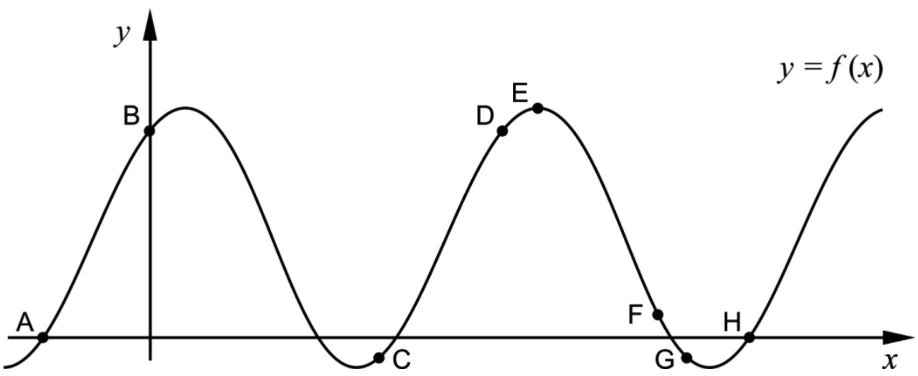
9. Simplify the expressions as far as possible.

a) $\dfrac{3x+15}{x+5}$ ______________________ (1/0/0)

b) $\dfrac{x^2-6x+9}{2x^2-18}$ ______________________ (0/1/0)

c) $\dfrac{(x-1)^{13}+(x-1)^{12}}{x}$ ______________________ (0/0/1)

10. The figure shows the graph of a function f. The points A – H are marked on the graph.



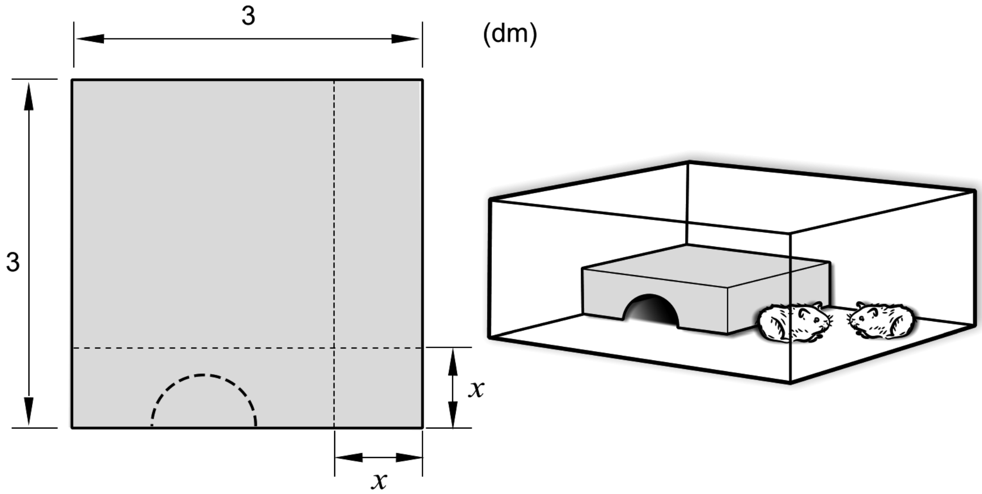
a) At one of the points A – H, $f'(x) > 0$ and $f(x) < 0$
Write down this point. ______________________ (0/1/0)

b) At several of the points A – H, $f''(x) < 0$
Write down these points. ______________________ (0/0/1)

Part C: Digital resources are not allowed. Do your solutions on separate sheets of paper.

11. The equation of a circle is $(x-3)^2+(y-2)^2=64$
Investigate if the point $(10,6)$ lies on the circle. (2/0/0)

12. János has a square piece of sheet metal which he will use to build a house for his hamsters. He is going to cut out a square from one of the corners of the sheet metal and then fold the sheet metal into a house, see figure.

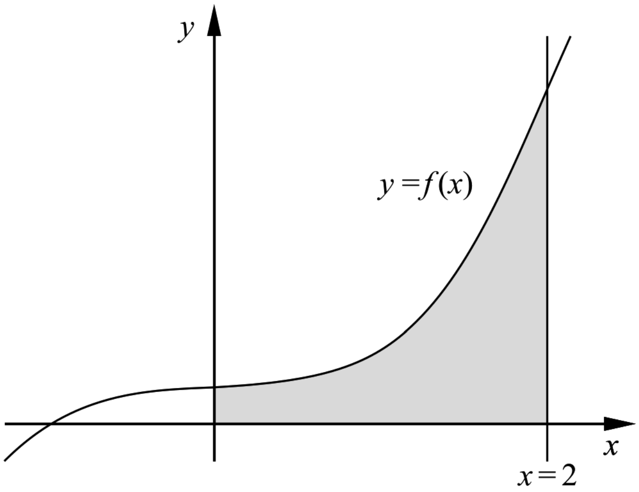


János assumes that the square has side x dm. He then determines the volume of the house $V\ \text{dm}^3$ as a function of the side x dm:

$$V(x)=x^3-6x^2+9x$$

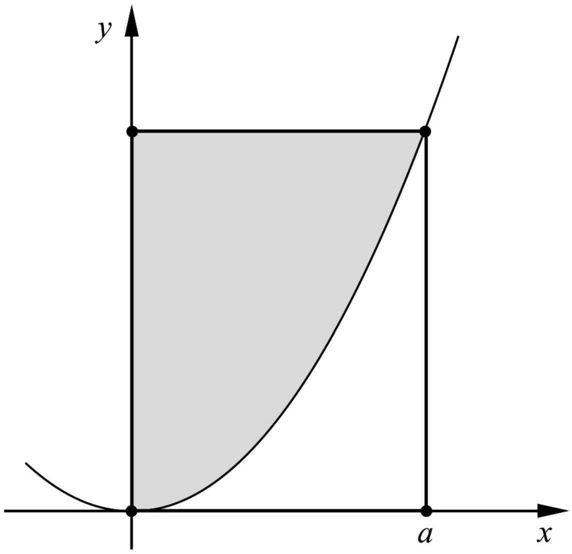
Use the derivative to calculate x so that the volume of the house becomes as large as possible. (3/1/0)

13. Calculate the area of the region bounded by the line $x = 2$, the graph of $f(x) = \dfrac{x^3 + 1}{4}$ and the positive coordinate axes. (0/2/0)



14. Archimedes was a Greek mathematician and philosopher who lived approximately 2300 years ago. He studied, among other things, parabolas.

The figure shows a parabola and a rectangle in a coordinate system. The rectangle has corners at the origin, on the parabola and at the positive coordinate axes. The parabola divides the rectangle into a grey region above the parabola and a white region below the parabola. See figure.



Archimedes claimed that the area of the grey region is twice the area of the white region.

Assume that the parabola is described by the function $y = kx^2$ where k is a positive constant and that the corner on the positive x-axis is at the point where $x = a$.

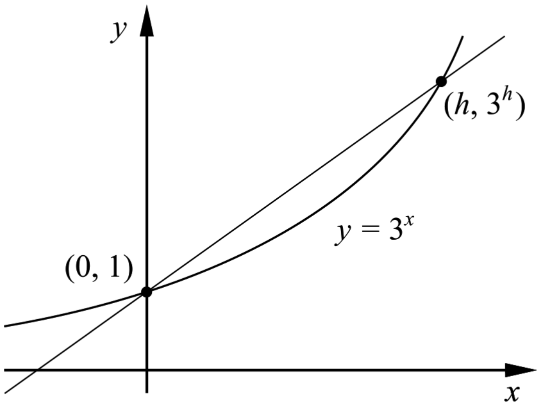
Prove that Archimedes' statement holds for all such parabolas. (0/3/0)

15. Determine all values of a so that it is possible to simplify the expression

$$\frac{x^2 - ax - 12}{x^2 + 2x - 3}$$

(0/0/2)

16. The figure shows the graph of $y = 3^x$ and a straight line that intersects the graph at the points $(0, 1)$ och $(h, 3^h)$.



Find $\lim\limits_{h \to 0} \dfrac{3^h - 1}{h}$. Give an exact answer. (0/0/2)