Part B	Problems 1–11 which only require answers.
Part C	Problems 12–18 which require complete solutions.
Test time	120 minutes for part B and part C together.
Resources	Formula sheet and ruler.

The test consists of three written parts (part B, C and D).
Together they give a total of 59 points consisting of 21 E-, 22 C- and 16 A-points.

Level requirements for test grades
E: 15 points
D: 24 points of which 7 points on at least C-level
C: 31 points of which 13 points on at least C-level
B: 39 points of which 5 points on A-level
A: 46 points of which 8 points on A-level

The number of points you can get for a complete solution is stated after each problem.
You can also see what knowledge levels (E, C and A) you can show in each problem.
For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled "*Only answer is required*" you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____

Date of birth: _____

Educational programme: _____

Part B: Digital tools are not allowed. *Only answer is required.* Write your answers in the test booklet.

1. One of the alternatives A–D is an example of a primitive function of the function $f(x) = x^3 - 2x$. Which one?
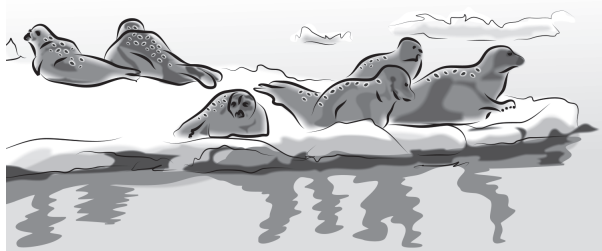
   A. $F(x) = 3x^2 - 2$

   B. $F(x) = \dfrac{x^4}{4} - 4x$

   C. $F(x) = \dfrac{x^4}{4} - x^2$

   D. $F(x) = x^4 - 2x^2$ _____ (1/0/0)

2. On August 1 each summer, the number of grey seals in the Baltic Sea is inventoried (counted). The table shows the result.
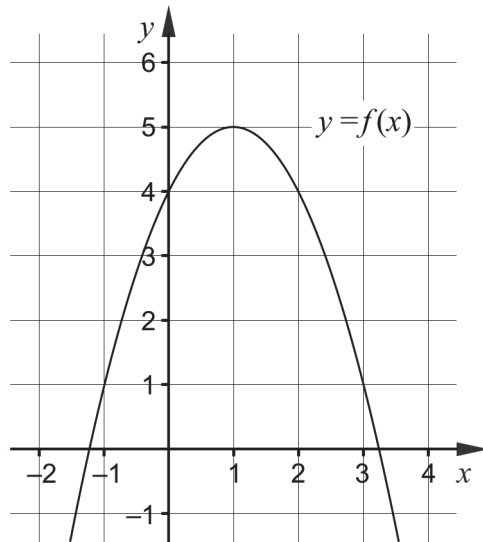
Year	Number of grey seals
2013	28 000
2014	32 000
2015	31 000
2016	30 000
2017	30 000
2018	34 000
2019	38 000

   Use the table and determine the average rate of change of the number of grey seals from August 1, 2015 to August 1, 2018.

   _____ seals/year (1/0/0)

3.   The figure shows the graph of the function f.

Use the graph and state which one of the alternatives A–F is the best value
for $f'(2)$.

A.   4

B.   2

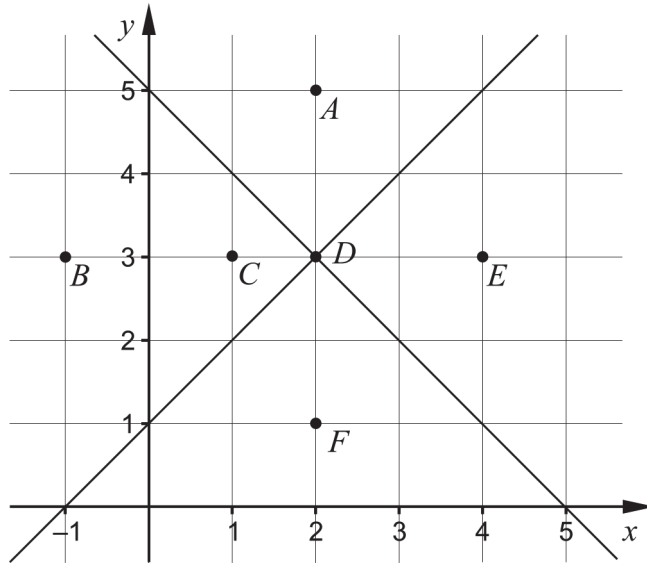C.   0.5

D.   −0.5

E.   −2

F.   −4   _____ (1/0/0)

4.   The 100 first numbers in a sequence form the geometric sum S,
where $S = \dfrac{2(5^{100} - 1)}{4}$

a)   Determine the first number in the sequence.   _____ (1/0/0)

b)   Determine the fourth number in the sequence.   _____ (0/1/0)

5. The figure shows two straight lines and six points A–F.

One of the points A–F lies in the region bounded by $\begin{cases} x > 0 \\ y < 5 - x \\ y > x + 1 \end{cases}$

Which one? _____ (1/0/0)

6. Determine $f'(x)$ when

a) $f(x) = 4x^3 - 12x$ $f'(x) =$ _____ (1/0/0)

b) $f(x) = ax^2 - \dfrac{4}{x}$ $f'(x) =$ _____ (0/1/0)

c) $f(x) = \dfrac{1}{3^{-2x}}$ $f'(x) =$ _____ (0/0/1)
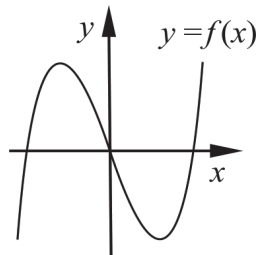
7. Solve the equation $3x^4 - 8x = 2x^4$ _____ (0/1/0)

8. Simplify the expressions as far as possible.
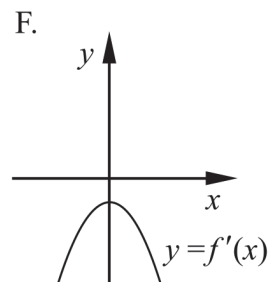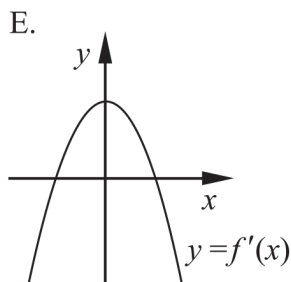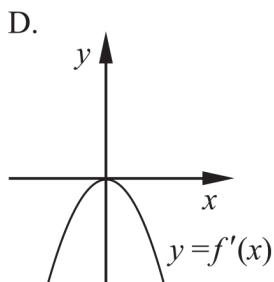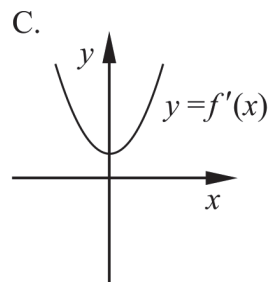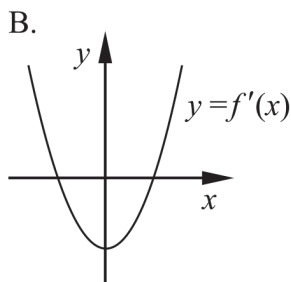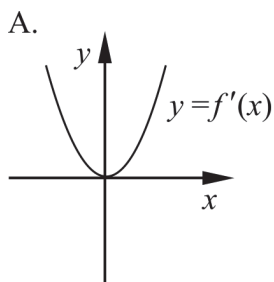
a) $\dfrac{5x^3 - x^6}{x^3}$ _____ (1/0/0)

b) $\dfrac{2x^2 + 12x + 18}{2(x^2 - 9)}$ _____ (0/1/0)

c) $\dfrac{2e^x \cdot e^{-ax} - e^x}{e^{-ax} - 0.5}$ _____ (0/0/1)

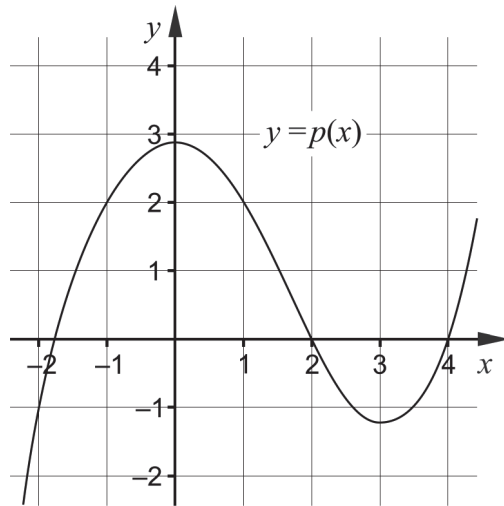**9.** The figure shows the graph of the function f.

One of the alternatives A–F shows the graph of the derivative f' of the function. Which one?

A.

B.

C.

D.

E.

F.

_____ (0/1/0)

10. The figure shows the main features of the graph of the function p.
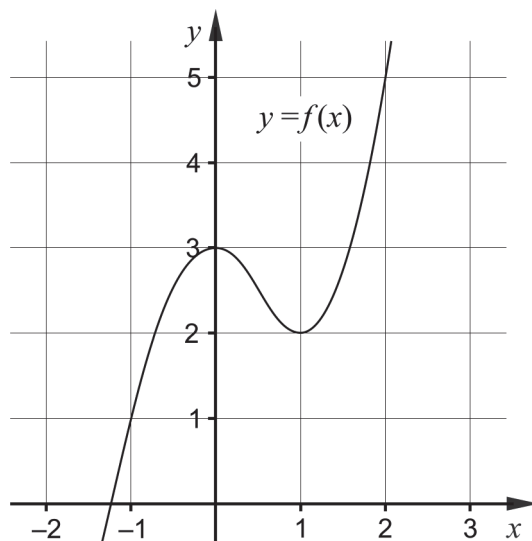

Determine for what values of x it holds that

a) $p'(x) < 0$ _____ (0/1/0)

b) the expression $\dfrac{p(x)}{p'(x)}$ is not defined. _____ (0/1/0)

11. The figure shows the graph of the function f.

Determine a value of a so that $\displaystyle\int_{-1}^{a} f'(x)\,dx = 3$ _____ (0/0/1)

Part C: Digital tools are not allowed. Write your solutions on separate sheets of paper.

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12. Tilde differentiates the function $f(x) = e^{2x}$ and writes down the ratio

$$\frac{f'(x)}{f(x)}$$

She claims the following: "For all values of x, the ratio will always have the value 2".

Is Tilde right? Justify your answer. (1/0/0)

13. Calculate $\displaystyle\int_{1}^{2} 3x^2 \, dx$. (2/0/0)

14. The function f is given by $f(x) = x^3 - 3x^2 + 7$
   Use derivatives and determine the coordinates of any maxima, minima or saddle points for the graph of the function.

    For each of the points also determine if it is a maximum, minimum or saddle point. (3/1/0)

15. The figure shows a shaded region bounded by the graph of the function g, the straight line $x = 3$ and the positive coordinate axes.
    The function g is given by $g(x) = 5 + px - x^2$ where p is a constant.
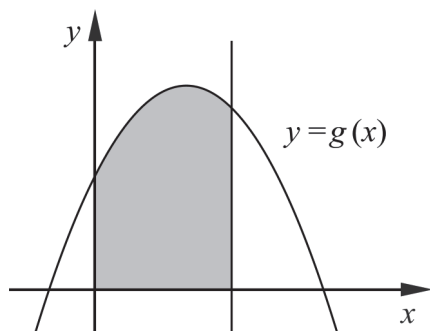
    Determine p so that the area of the shaded region is 24 area units. (0/2/0)

16. The function f is given by $f(x) = x^3 + 3x$

Jaana claims that the function f has two extremal points.
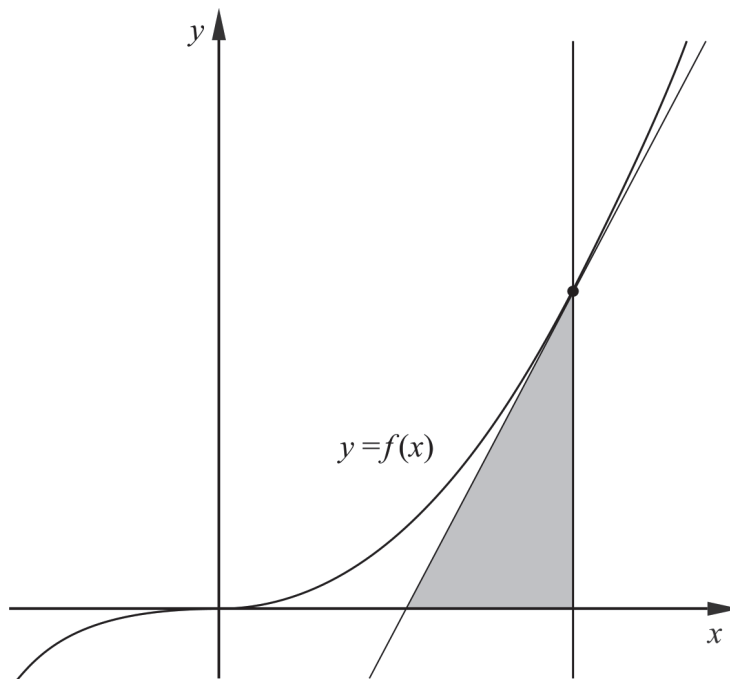
Is Jaana right? Justify your answer. (0/2/0)

17. The function f is given by $f(x) = \dfrac{5}{a^2 x}$ where $x \neq 0$ and $a \neq 0$

Determine $f'(x)$ using the definition of the derivative. (0/1/3)

18. The figure shows the graph of the cubic function f given by $f(x) = x^3$ and a tangent to the graph in the point where $x = a$. The tangent, the positive x-axis and the line $x = a$ bound a region shaped like a triangle.

Determine a so that the triangle has area 1.5 area units. (0/0/3)